Magnetoresistance due to edge spin accumulation

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Because of spin-orbit interaction, an electrical current is accompanied by a spin current resulting in spin accumulation near the sample edges. Due again to spin-orbit interaction this causes a small decrease of the sample resistance. An applied magnetic field will destroy the edge spin polarization leading to a positive magnetoresistance. This effect provides means to study spin accumulation by electrical measurements. The origin and the general properties of the phenomenological equations describing coupling between charge and spin currents are also discussed.

It was predicted a long time ago [1, 2] that because of spin-orbit interaction electrical and spin currents are interconnected: an electrical current produces a transverse spin current and *vice versa*. In recent years this has become a subject of considerable interest.

The purpose of this Letter is twofold. First, another way of understanding this interconnection will be presented and some general properties of the resulting phenomenological equations will be discussed. Second, a new magnetoresistance effect will be considered, which allows to study the current-induced spin accumulation near the sample edges by purely electric measurements.

The transport phenomena related to coupling of the spin and charge currents can be described phenomenologicaly in the following simple way. Let $\boldsymbol{q}$ be the electron flow density and let $\boldsymbol{q}^{(0)}$ be its conventional expression not accounting for spin-orbit interaction:

$$\boldsymbol{q}^{(\mathbf{0})} = -\mu n\boldsymbol{E} - D\nabla n, \tag{1}$$

where μ and D are the usual electron mobility and diffusion coefficient, connected by the Einstein relation, $\boldsymbol{E}$ is the electric field, and n is the electron concentration. The electric current density is $\boldsymbol{j} = -e\boldsymbol{q}$, where e is the absolute value of the electron charge.

Let q_{ij} be the spin polarization current density tensor (the flow of the j component of the spin polarization in the direction i). It should be understood that for polarized electrons the spin current may exist even in the absence of spin-orbit interaction, simply because spins are carried by electron flow. We denote the corresponding quantity as $q_{ij}^{(0)}$. Then, similar to Eq. (1), we have

$$q_{ij}^{(0)} = -\mu E_i P_j - D\frac{\partial P_j}{\partial x_i}, \tag{2}$$

where $\boldsymbol{P}$ is the vector of electron spin polarization density. If there are other sources for currents, like for example a temperature gradient, the corresponding terms should be included in Eqs. (1) and (2).

We have departed from the conventional definitions [1, 2] by introducing the vector of spin *polarization* density $\boldsymbol{P}$ and the spin *polarization* current q_{ij}. This allows to avoid numerous factors 1/2 and 2 in the formulas to follow. One can return to the traditional notations by putting $\boldsymbol{P} = 2\boldsymbol{S}$, where $\boldsymbol{S}$ is the spin density, and by replacing q_{ij} by $q_{ij}/2$ to obtain the true spin current density.

Spin-orbit interaction couples the two currents. For a material with inversion symmetry [3] we have:

$$q_i = q_i^{(0)} + \gamma\epsilon_{ijk}q_{jk}^{(0)}, \tag{3}$$

$$q_{ij} = q_{jk}^{(0)} - \gamma\epsilon_{ijk}q_k^{(0)}, \tag{4}$$

where ϵ_{ijk} is the unit antisymmetric tensor and γ is a dimensionless coupling constant proportional to the spin-orbit interaction, it is assumed that $\gamma << 1$. The difference in signs in Eqs. (3) and (4) is consistent with the Onsager relations and is due to the different properties of $\boldsymbol{q}$ and q_{ij} with respect to time inversion [4].

Explicit phenomenological expressions for the two currents follow from Eqs. (1)-(4):

$$\boldsymbol{j}/e = \mu n\boldsymbol{E} + D\nabla n + \beta\boldsymbol{E}\wedge\boldsymbol{P} + \delta\,\mathrm{curl}\,\boldsymbol{P}, \tag{5}$$

$$q_{ij} = -\mu E_i P_j - D\frac{\partial P_j}{\partial x_i} + \epsilon_{ijk}(\beta n E_k + \delta\frac{\partial n}{\partial x_k}). \tag{6}$$

Here

$$\beta = \gamma\mu, \qquad \delta = \gamma D, \tag{7}$$

so that the coefficients β and δ, similar to μ and D, satisfy the Einstein relation.

Eqs. (5) and (6) should be complemented by the equation for the spin polarization vector:

$$\frac{\partial P_j}{\partial t} + \frac{\partial q_{ij}}{\partial x_i} + (\boldsymbol{\Omega}\wedge\boldsymbol{P})_j + \frac{P_j}{\tau_s} = 0, \tag{8}$$

where the vector $\boldsymbol{\Omega}$ is directed along the applied magnetic field, Ω being the spin precession frequency and τ_s is the spin relaxation time. In Eqs. (6), (7) we ignore the action of magnetic field on the particle dynamics. This is justified if $\omega_c\tau << 1$, where ω_c is the cyclotron frequency and τ is the momentum relaxation time. Since normally $\tau_s >> \tau$, it is possible to have both $\Omega\tau_s >> 1$ and $\omega_c\tau << 1$ in a certain interval of magnetic fields. It is also assumed that the equilibrium spin polarization in the applied magnetic field is negligible.

While Eqs. (5)-(8) are written for a three-dimensional sample, they are equally applicable to the 2D case, with obvious modifications: the electric field, space gradients,

and all currents (but not the spin polarization vector) should have components in the 2D plane only.

In the equilibrium situation all currents should obviously vanish. If an inhomogeneous magnetic field exists, the equilibrium spin polarization will be space-dependent, however this by itself should produce neither spin, nor charge currents. To assure this, an additional counter-term should be introduced into the right-hand side of Eq. (2), proportional to $\partial B_j/\partial x_i$, which takes care of the force acting on the electron with a given spin in an inhomogeneous magnetic field $\boldsymbol{B}(\boldsymbol{r})$ (see [5]). Corresponding terms will appear in Eqs. (5), (6). We ignore these terms assuming that $\boldsymbol{B}$ is homogeneous.

Equations (5)-(8), which appeared for the first time in Refs. [1, 2] describe all the physical consequences of spin-charge current coupling [6]. The term $\beta \boldsymbol{E} \wedge \boldsymbol{P}$ describes the anomalous Hall effect [7], where the spin polarization plays the role of the magnetic field.

The term $\delta\,\mathrm{curl}\,\boldsymbol{P}$ describes an electrical current induced by an inhomogeneous spin density (now referred to as the Inverse Spin Hall Effect). A way to measure this current under the conditions of optical spin orientation was proposed in [8]. The circularly polarized exciting light is absorbed in a thin layer near the surface of the sample. As a consequence, the photo-created electron spin density is inhomogeneous, however $\mathrm{curl}\,\boldsymbol{P} = 0$, since both $\boldsymbol{P}$ and its gradient are perpendicular to the surface. By applying a magnetic field parallel to the surface one can create a parallel component of $\boldsymbol{P}$, thus inducing a non-zero $\mathrm{curl}\,\boldsymbol{P}$ and the corresponding surface electric current (or voltage). This effect was found experimentally for the first time by Bakun *et al* [9].

The term $\beta n \epsilon_{ijk} E_k$ (and its diffusive counterpart $\delta \epsilon_{ijk} \partial n/\partial x_k$) in Eq. (6), describes what is now called the Spin Hall Effect: an electrical current induces a transverse spin current, resulting in spin accumulation near the sample boundaries [1, 2]. This phenomenon was observed experimentally only in recent years [10, 11] and has attracted widespread interest.

It should be stressed that all these phenomena are closely related and have their common origin in the coupling between spin and charge currents given by Eqs. (3) and (4). Any mechanism that produces the anomalous Hall effect will also lead to the spin Hall effect and *vice versa*. It is remarkable that there is a single dimensionless parameter, γ, that governs the resulting physics. The calculation of this parameter should be the objective of a microscopic theory. For the case, when the coupling is due to spin asymmetry in electron scattering, this was done in Ref. [2], where β and δ were expressed through the scattering amplitude. In this case γ depends only on the form of the scattering potential, the electron energy, and the strength of spin-orbit interaction.

An "intrinsic" mechanism of the spin Hall effect, related only to spin band splitting, was proposed for bulk holes in the valence band [12]. The value of γ is on the order of $(k_F \ell)^{-1}$ (k_F is the Fermi wavevector, ℓ is the mean free path) and generally depends on the details of the scattering mechanism. The current consensus is that the intrinsic mechanism may exist for any type of spin band splitting, *except* if it is linear in k [13].

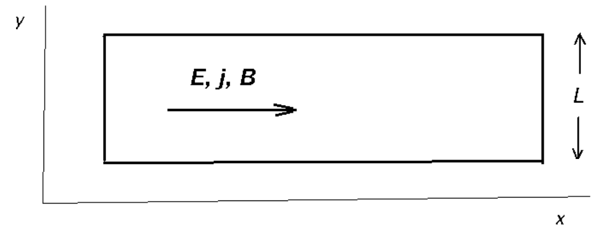

FIG. 1: The geometry of the proposed experiment with a 2D sample. The direction of the magnetic field is of no importance, so long as it lies in the 2D plane. For simplicity it is assumed to be parallel to the current.

Note that the $J = 3/2$ holes may not be described by the simple Eqs. (5)-(8), because for higher spins the number of coupled macroscopic quantities increases compared to spin 1/2 particles. The mutual transformation of spin and charge currents for holes, due to scattering, was studied in [14]. Also, even in the absence of spin-orbit interaction, holes are still particles with internal angular momentum $L = 1$ and the splitting into light and heavy holes still exists. Thus, for the case of holes the spin-orbit interaction is not of primary importance.

We now discuss a new related phenomenon: a magnetoresistance due specifically to spin accumulation near the sample edges. Since the accumulation occurs on the scale of the spin diffusion length $L_s = \sqrt{D\tau_s}$ (the "spin layer" [1]), the proposed effect depends on the sample size, L, and becomes negligible when $L >> L_s$.

Whithin the spin layer the z component of spin polarization changes in the direction perpendicular to the sample boundary (the y direction). Thus $\mathrm{curl}\,\boldsymbol{P} \neq 0$, and according to Eq. (5) a correction to the electric current should exist. As we will see, this correction is positive, i.e. it leads to a slight decrease of the sample resistance compared to the (hypothetical) case when spin-orbit interaction is absent. By applying a magnetic field in the xy plane, we can destroy the spin polarization (the Hanle effect) and thus observe a positive magnetoresistance on a field scale corresponding to $\Omega\tau_s \sim 1$. One might say that this is a manifestation of combined direct and inverse spin Hall effects, and the Hanle effect.

We will consider a 2D sample (see Fig. 1), a similar effect will exist also for a thin wire. The advantage of the 2D case is that the small effect considered here will not be masked by the normal magnetoresistance, because the magnetic field parallel to the 2D plane acts on the spins only, but not on the electron orbital motion. Since the spin polarization is proportional to the electric field, we discard nonlinear in E terms proportional to EP.

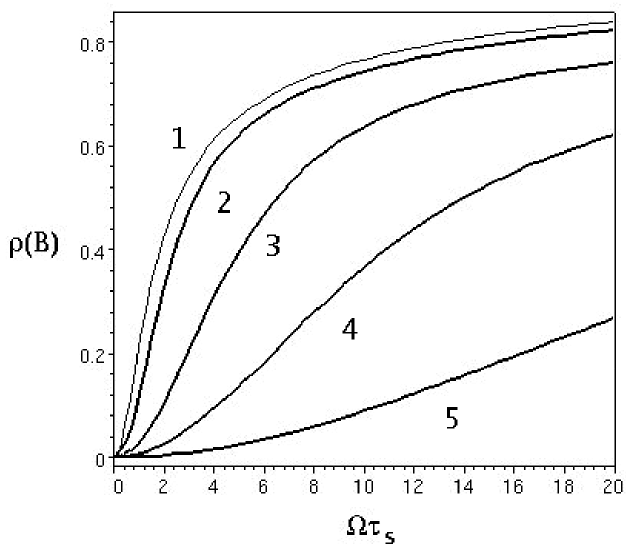

FIG. 2: Normalized magnetoresistance as a function of the parameter $\Omega\tau_s$ for different sample widths. *1* - Eq. (18) for $\lambda >> 1$, *2* - $\lambda = 1.5$, *3* - $\lambda = 0.8$, *4* - $\lambda = 0.5$, *5* - $\lambda = 0.3$

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For the geometry of Fig. 1, from Eq. (5) we obtain:

$$j = e(\mu nE + \delta \frac{dP_z}{dy}), \tag{9}$$

so that the total current is

$$I = \int_{-L/2}^{L/2} j(y)dy = I_0 + \Delta I, \tag{10}$$

where

$$I_0 = e\mu nEL, \quad \Delta I = e\delta(P_z(L/2) - P_z(-L/2)). \tag{11}$$

The correction to the current, ΔI, is proportional to the difference in spin polarization at the opposite edges of the sample. Eq. (6) yealds:

$$q_{yz} = -D\frac{dP_z}{dy} + \beta nE, \qquad q_{yy} = -D\frac{dP_y}{dy}. \tag{12}$$

In the steady state Eq. (8) gives:

$$D\frac{d^2P_z}{dy^2} - \Omega P_y = \frac{P_z}{\tau_s}, \quad D\frac{d^2P_y}{dy^2} + \Omega P_z = \frac{P_y}{\tau_s}. \tag{13}$$

These equations should be solved with the boundary conditions at $y = \pm L/2$:

$$\frac{dP_z}{dy} = \frac{\beta nE}{D}, \qquad \frac{dP_y}{dy} = 0, \tag{14}$$

corresponding to vanishing spin currents q_{yz} and q_{yy} at the sample edges.

A straightforward calculation gives the result:

$$\frac{\Delta R}{R_0} = -\frac{\Delta I}{I_0} = -\gamma^2 \mathrm{Re}\Big[\frac{\tanh(\kappa\lambda)}{\kappa\lambda}\Big], \tag{15}$$

where R_0 is the uncorrected sample resistance, ΔR is the field-dependent negative correction due to spin accumulation,

$$\kappa = (1 - ix)^{1/2}, \qquad x = \Omega\tau_s, \qquad \lambda = L/(2L_s),$$

and $L_s = (D\tau_s)^{1/2}$ is the spin diffusion length.

Thus ΔR is proportional to the square of the dimensionless parameter γ in Eqs. (3), (4). In deriving Eq. (15) the relations given by Eq. (7) were used.

From this result one can easily deduce two characteristic features of this effect.

1) *The total resistance change* between its zero-field value, $R(0)$ and its value at strong enough field ($\Omega\tau_s >> 1$), $R(\infty) = R_0$:

$$\frac{R(\infty) - R(0)}{R(0)} = \gamma^2 \frac{\tanh\lambda}{\lambda}, \qquad (16).$$

For a narrow sample, $\lambda << 1$ the overall relative change of resistance is equal to γ^2, which gives a nice way to determine experimentally the fundamental parameter γ. For wide samples the relative change is $\gamma^2(2L_s/L)$.

2) *The shape of the magnetoresistance curve.* We introduce the notation $\rho(B)$ for the normalized relative magnetoresistance. Then

$$\rho(B) = \frac{R(B) - R(0)}{R(\infty) - R(0)} = 1 - \mathrm{Re}\Big[\frac{\tanh(\kappa\lambda)}{\kappa\tanh\lambda}\Big], \tag{17}$$

For a wide sample, $\lambda >> 1$, the width of the magnetoresistance curve is determined by the condition $\Omega\tau_s \sim 1$). In this case

$$\rho(B) = 1 - \mathrm{Re}\Big(\frac{1}{\kappa}\Big) = 1 - \Big[\frac{1 + \sqrt{1 + x^2}}{2(1 + x^2)}\Big]^{1/2}. \tag{18}$$

Note that at $x = \Omega\tau_s >> 1$ the function $\rho(B)$ approaches its maximum value very slowly, as $1 - 1/\sqrt{2x}$.

Figure 2 presents numerical results for $\rho(B)$ calculated from Eq. (17) for different values of $\lambda = L/(2L_s)$ together with the curve given by Eq. (18) for $\lambda >> 1$, which is a good approximation already for $\lambda = 1.5$.

For narrow samples ($\lambda << 1$) the magnetic field dependence becomes much weaker. The reason is that along with the spin relaxation time τ_s, there is another characteristic time, $\tau_d = \tau_s\lambda^2 = L^2/(4D)$, which is the time of diffusion on a distance $L/2$. For narrow samples ($\tau_d < \tau_s$), it is this time, rather than τ_s, that determines the width of the Hanle curve, because the spin polarization is destroyed by diffusion faster than by spin relaxation. Accordingly, the width of the magnetoresistance curve will now correspond to $\Omega\tau_d \sim 1$, i.e. it will be $1/\lambda^2$ times broader compared to the case of a wide sample.

To unify the two limiting cases, in Fig. 3 we re-plot $\rho(B)$ as a function of the parameter $\Omega\tau^*$, where τ^* is the effective time during which the spin is destroyed because of the combined effect of spin diffusion and spin relaxation:

$$\frac{1}{\tau^*} = \frac{1}{\tau_s} + \frac{1}{\tau_d} = \frac{1}{\tau_s}(1 + \frac{1}{\lambda^2}).$$

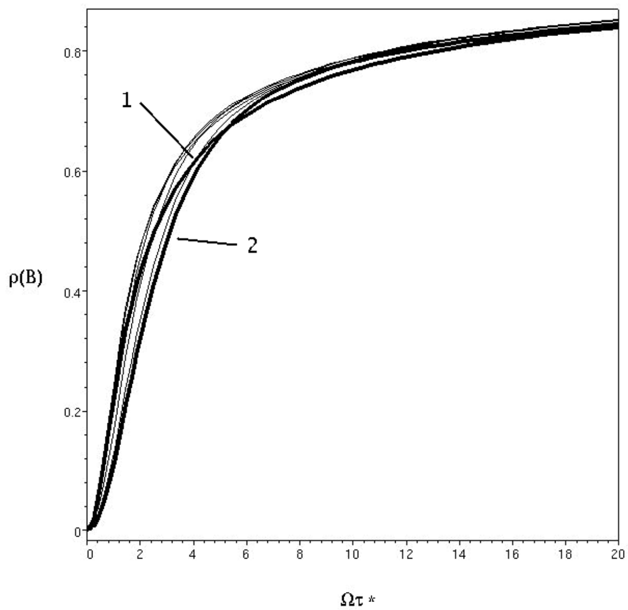

FIG. 3: Normalized magnetoresistance as a function of the parameter $\Omega\tau^*$, where $1/\tau^* = 1/\tau_s + 1/\tau_d$. *1* - in the limit $\lambda >> 1$, *2* - in the limit $\lambda << 1$. Other curves correspond to $\lambda = 0.4$, 0.8, 1.2, 1.6, and 2. One can see that all curves practically coincide.

Figure 3 shows that as a function of this parameter there is a quasi-universal curve, since the results for the limiting cases of a narrow and a wide samples are very close. Thus Eq. (18) can serve as a good interpolation formula for the general case, provided the variable x is replaced by $\Omega\tau^*$, instead of $\Omega\tau_s$. The high-field limit is always approached as $1/\sqrt{B}$.

The above results for magnetoresistance are similar to those obtained previously [15] for the Hanle effect in the case when spin polarization is inhomogeneous and spin diffusion is important.

From the experimental results for 3D [10] and 2D [5] GaAs one can estimate $\gamma \sim 10^{-2}$. Kimura *et al* [16] find $\gamma = 3.7 \cdot 10^{-3}$ for Platinum at room temperature, so that in these two cases a magnetoresistance due to spin accumulation on the order of 10^{-4} and 10^{-5}, respectively, can be expected. The characteristic feature, allowing to identify this effect, is the specific form of the magnetoresistance curve, as well as its strong dependence on the sample width, when it becomes comparable to the spin diffusion length.

Because of the high precision of electrical measurements, magnetoresistance might provide a useful tool for studying the spin-charge interplay in semiconductors and metals.

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[3] In the absence of inversion symmetry there may be additional terms describing this coupling. In particular, there is a spin current induced by a non-equilibrium spin polarization and a uniform spin polarization generated by electric current.

[4] A simple way to verify these equations is to start with currents $\boldsymbol{q}^\pm$ for particles with spin-up and spin-down (with respect to the z axis), assuming that the electric field and the concentration gradient are along x: $q_x^\pm = -\mu n_\pm E - D\partial n_\pm/\partial x$. (Spin-up and spin-down mobilities are taken equal, which is true when the spin polarization is small). Because of spin-orbit interaction these currents will induce currents of opposite signs in the y-direction: $q_y^\pm = \mp\gamma q_x^\pm$. It can now be seen that the expressions for $q_y = q_y^+ + q_y^-$ and $q_{yz} = q_y^+ - q_y^-$ coincide with what is given by Eqs. (1)-(4) with $P_z(x) = n_+ - n_-$. If, instead of the spin *polarization* current, the normal spin current were used, we would have to replace the coefficient γ by 2γ in Eq.(3) and by $\gamma/2$ in Eq.(4).

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[6] In [1, 2] these equations were written in terms of spin density $\boldsymbol{S}$. Thus the definitions of the coefficients in Eqs. (5), (6), and the relations between them, were different (by numerical factors) from those adopted here.

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[13] However, it does not seem reasonable to introduce "extrinsic" and "intrinsic" spin Hall effects as distinct physical phenomena. The phenomenon is always the same: current induced edge spin accumulation, while certainly there may be different microscopic mechanisms for spin-charge current coupling.

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